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Exact Results and Duality for $SP(2N)$ SUSY Gauge Theories with an Antisymmetric Tensor

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Abstract

We study supersymmetric $Sp(2N)$ gauge theories with matter in the antisymmetric tensor representation and F fundamentals. For $F = 6$ we solve the theory exactly in terms of confined degrees of freedom and a superpotential. By adding mass terms we obtain the theories with $F < 6$ which we find to exhibit a host of interesting non-perturbative phenomena: quantum deformed moduli spaces with N constraints, instanton-induced superpotentials and non-equivalent disjoint branches of moduli spaces. We find a simple dual for $F = 8$ and no superpotential. We show how the $F = 4$ and $F = 2$ theories can be modified to break supersymmetry spontaneously and point out that the $Sp(6)$ theory with $F = 6$ may be very interesting for model builders.

1 Introduction

There is an increasing number of asymptotically-free $N = 1$ supersymmetric (SUSY) gauge theories that can be analyzed non-perturbatively using techniques recently discovered by Seiberg [1, 2]. These theories exhibit many interesting phenomena generated by the non-perturbative dynamics.

The "classic" example is SUSY QCD: an $SU(N_c)$ gauge theory with N_f fields in fundamental and antifundamental representations. For $N_f < N_c$, there is a dynamically generated superpotential, which is singular at the origin [3]. For $N_f = N_c$, the infrared theory confines and the classical constraint on the meson and baryon fields is modified quantum mechanically, resulting in chiral symmetry breaking [1]. For $N_f = N_c + 1$, the low-energy theory is confining without chiral symmetry breaking [1]. For $N_c + 1 < N_f < 3N_c$, the theory has a dual description in terms of an $SU(N_f - N_c)$ gauge theory [2]. The two descriptions are completely equivalent in the infrared.

A similar analysis can be carried out for other gauge groups. $SO(N)$ and $Sp(2N)$ theories have been analyzed in Refs. [2, 4] and Refs. [2, 5], respectively. Theories with more complicated matter have been analyzed as well. Examples include $SU(N)$ theories with an antisymmetric tensor [6, 7] and $SU(N)$, $SO(N)$ and $Sp(2N)$ theories including matter in the adjoint representation [8, 9]. There are also similar results for product groups [10, 11].

In this paper, we analyze $Sp(2N)$ gauge theories containing one antisymmetric tensor in addition to F fields in the fundamental representation. We find that $Sp(2N)$ theories for $F = 6$ confine without breaking chiral symmetries and can be described by an exact superpotential. We identify the low-energy spectrum and derive the superpotential. Using this superpotential we obtain the superpotential for $F < 6$ by integrating out matter. For $F = 4$, we find that the theory is still confining, but the moduli space is constrained. One of the classical constraints is modified quantum mechanically, while the remaining $N - 1$ constraints are unmodified. By integrating out more matter, we find the dynamically generated superpotentials for $F = 0, 2$.

For $F = 8$, we obtain a simple dual for the theory without the addition of a superpotential. This dual has the same $Sp(2N)$ gauge group and contains extra gauge singlet meson fields and a superpotential. This is the first example in the literature of a simple dual of a theory containing a two-index tensor and no superpotential. However, it seems very difficult to extend this duality for $F > 8$. After addition of a superpotential that breaks some of the

global symmetries and simplifies the infrared theory by lifting some of the classical flat directions, a dual description can be found for arbitrary F [12].

These results about $Sp(2N)$ can have several important applications to model building. The fact that $Sp(6)$ with $F = 6$ has three antisymmetric tensors in its low-energy description can be used for building of composite models. We present a simple toy model based on $Sp(6)$ that naturally generates a hierarchy for fermion masses. Our results can also be applied to construct models of dynamical supersymmetry breaking. We give one example of supersymmetry breaking by an instanton-generated superpotential and one example with a quantum-deformed moduli space.

This paper is organized as follows: in the next section, we solve the theories for $F \leq 6$ and present consistency checks on our solutions. Section 3 is devoted to the theories with $F > 6$. Section 4 contains applications of our results to dynamical supersymmetry breaking and model building. Finally, we conclude in Section 5.

2   Confinement and Exact Superpotentials for $F \leq 6$

We consider supersymmetric $Sp(2N)$ gauge theories containing an antisymmetric tensor A, and F fields Q_i in the fundamental representation. The Witten anomaly requires that F is even [13].

The non-anomalous global symmetry of the microscopic theory is $SU(F) \times U(1) \times U(1)_R$, with one possible charge assignment given in the table below.

$$
\begin{array}{c|cccc}
 & Sp(2N) & SU(F) & U(1) & U(1)_R \\
\hline
A & \square\!\square & 1 & -\frac{F}{2} & 0 \\
Q_i & \square & \square & N-1 & 1-\frac{4}{F}
\end{array}
\tag{1}
$$

Note that for $F = 4$ one can define an anomaly free R-symmetry under which all chiral superfields carry charge zero. In analogy with SUSY QCD, we expect that this theory might confine and have a quantum deformed moduli space. Furthermore, we then expect that the theory with $F = 6$ confines without chiral symmetry breaking and that the proper degrees of freedom in the infrared are given by the set of independent gauge-invariant

operators

$$T_k = \frac{1}{4k}\mathrm{Tr}A^k, \ k = 2, 3, \ldots, N, \tag{2}$$

$$M_k = QA^kQ, \ k = 0, 1, 2, \ldots, N - 1, \tag{3}$$

where all contractions are performed using the $Sp(2N)$ invariant antisymmetric tensor $J = i\sigma_2 \otimes 1_{N\times N}$. The symmetry properties of these fields for $F = 6$ are given in the table below:

	$SU(6)$	$U(1)$	$U(1)_R$
T_k	1	$-3k$	0
M_k	$\boxminus$	$2(N-1) - 3k$	$\frac{2}{3}$

$$\tag{4}$$

It is a very non-trivial consistency check that for $F = 6$ the global anomalies of the ultraviolet theory are matched by these gauge invariants. The global anomalies in both microscopic and macroscopic descriptions are

$$
\begin{aligned}
SU(6)^3 &= 2N \\
SU(6)^2 U(1) &= 2N(N-1) \\
SU(6)^2 U(1)_R &= -\frac{4}{3}N \\
U(1)^3 &= -27[N(2N-1)-1] + 12N(N-1)^3 \\
U(1)^2 U(1)_R &= (1-N)(8N^2 + 10N + 9) \\
U(1)_R^2 U(1) &= \frac{1}{3}(1-N)(2N+9) \\
U(1)_R^3 &= 1 - \frac{14}{9}N \\
U(1) &= 3(N-1)(2N-1) \\
U(1)_R &= 1 - 6N
\end{aligned}
\tag{5}
$$

We believe that, technically, this is the most non-trivial example of anomaly matching in the literature; it involves the use of the identities: $\sum_1^n k^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_1^n k^3 = \frac{n^2(n+1)^2}{4}$. The fact that one can match the anomalies with this set of operators strongly suggests that, as expected, the $F = 6$ theories are in a phase of confinement without chiral symmetry breaking, where the low-energy degrees of freedom are given by the M's and T's above.

3

Using symmetry arguments and demanding that the equations of motion reproduce the classical constraints uniquely determine the non-perturbative superpotential in the gauge-invariant fields. Since the number of terms in the superpotential grows rapidly with N we only list the superpotentials for $N = 2, 3, 4$:

$$
\begin{aligned}
W_{F=6}^{Sp(4)} &= \frac{1}{\Lambda_{Sp(4)}^5}\left(\frac{1}{3}T_2 M_0^3 + \frac{1}{2}M_0 M_1^2\right) \\
W_{F=6}^{Sp(6)} &= \frac{1}{\Lambda_{Sp(6)}^7}\left(\frac{1}{3}T_2^2 M_0^3 + \frac{1}{2}T_3 M_1 M_0^2 - \frac{1}{2}T_2 M_0^2 M_2 + \right. \\
&\quad \left. \frac{1}{4}M_0 M_2^2 + \frac{1}{4}M_1^2 M_2\right) \\
W_{F=6}^{Sp(8)} &= \frac{1}{\Lambda_{Sp(8)}^9}\left(\frac{1}{3}T_2^3 M_0^3 + \frac{1}{3}T_3^2 M_0^3 - \frac{1}{3}T_2 T_4 M_0^3 + T_2 T_3 M_0^2 M_1 + \right. \\
&\quad \frac{1}{2}T_4 M_0 M_1^2 + \frac{1}{2}T_4 M_0^2 M_2 + \frac{1}{2}T_2^2 M_0 M_1^2 - \frac{1}{2}T_2^2 M_2 M_0^2 + \frac{1}{6}T_3 M_1^3 - \\
&\quad \frac{1}{2}T_3 M_0^2 M_3 - T_2 M_0 M_1 M_3 - \frac{1}{2}T_2 M_1^2 M_2 + \frac{1}{4}M_0 M_3^2 + \\
&\quad \left. \frac{1}{2}M_1 M_2 M_3 + \frac{1}{12}M_2^3\right),
\end{aligned}
\tag{6}
$$

where the $SU(6)$ flavor indices are contracted with $M^3 \equiv \epsilon^{ijklmn}M_{ij}M_{kl}M_{mn}$. Note that the terms $T_2 M_0 M_1^2$ in $Sp(6)$ and $T_3 M_0 M_1 M_2$ and $T_2 M_0 M_2^2$ in $Sp(8)$ are allowed by the symmetries but their coefficients vanish.

We now present further non-trivial consistency checks which corroborate the results for the superpotentials in Eq. 6. First, we add a mass term mT_2 for the antisymmetric tensor in the $Sp(4)$ theory. This way we obtain a theory with six fundamentals and no antisymmetric tensor, which is known to be in the confining phase with chiral symmetry breaking [5]. The new superpotential of our theory is

$$
W = T_2\left(m + \frac{1}{\Lambda^5}M_0^3\right) + \frac{1}{\Lambda^5}M_0 M_1^2.
\tag{7}
$$

The T_2 equation of motion $M_0^3 = m\Lambda^5 = \tilde{\Lambda}^6$ forces non-zero vacuum expectation values for M_0. This vacuum expectation value renders all components of M_1 massive via the $M_0 M_1^2$ term in the superpotential. Furthermore, T_2 pairs up to get a mass with the component of M_0 that lies in the direction

of the constraint $M_0^3 = \tilde{\Lambda}^6$. Thus, we reproduce the results of Ref. [5] for an $Sp(4)$ theory with six fundamentals as required by consistency.

Next, consider breaking $Sp(4)$ to $SU(2)$ by giving vevs to Q_5 and Q_6. In the ultraviolet theory two of the resulting $SU(2)$ doublets are eaten by the Higgs mechanism. Six doublets remain, four from $Q_{1,2,3,4}$ and two from A. To see that our infrared theory reproduces the correct result for an $SU(2)$ theory with six fundamentals, we substitute the vevs into the $Sp(4)$ superpotential of Eq. 6 to obtain the superpotential $W = \frac{1}{\Lambda^3}\text{Pf}M$. This is indeed the correct superpotential for the $SU(2)$ theory with six doublets if M is identified with the $SU(2)$ meson matrix.

Another way of connecting our results to a known theory is to break $Sp(2N)$ to $SU(2)^N$ by giving a vev of the following form to A:

$$\langle A \rangle = i\sigma_2 v \otimes \begin{pmatrix} \omega_1 & & & \\ & \omega_2 & & \\ & & \ddots & \\ & & & \omega_N \end{pmatrix}, \tag{8}$$

where ω_i are the N-th roots of 1. Note that this vev is traceless as required. The scales of the $SU(2)$ theories are related to the scale of the $Sp(2N)$ theory by $\Lambda_{Sp(2N)}^{2N+1} = \Lambda_{SU(2)_k}^3 v^{2(N-1)} w_{-2k}$. Taking the limit $v \to \infty$ and $\Lambda_{Sp(2N)} \to \infty$ while holding the $\Lambda_{SU(2)}$'s fixed we obtain a product of N decoupled $SU(2)$ theories with six fundamentals each. In the infrared theory we obtain the operator mapping from $Sp(2N)$ to $SU(2)^N$ by scaling out the large vevs from the T's and M's:

$$\begin{aligned}
M_0 &\to N_1 + N_2 + \ldots + N_N \\
M_1 &\to (\omega_1 N_1 + \omega_2 N_2 + \ldots + \omega_N N_N)v \\
M_2 &\to (\omega_2 N_1 + \omega_4 N_2 + \ldots + \omega_{2N} N_N)v^2 \\
&\vdots \\
M_{N-1} &\to (\omega_{N-1} N_1 + \omega_{2(N-1)} N_2 + \ldots + \omega_{N(N-1)} N_N)v^{N-1} \\
\langle T_2 \rangle &= \langle T_3 \rangle = \ldots = \langle T_{N-1} \rangle = 0 \\
\langle T_N \rangle &= Nv^N,
\end{aligned} \tag{9}$$

where N_i is the meson field of $SU(2)_i$. Substituting these expressions together with the scale matching relations into the superpotentials of Eq. 6, we

reproduce the correct superpotentials for the N decoupled $SU(2)$ theories

$$W = \frac{1}{\Lambda^3_{SU(2)}} \left(N^3_1 + N^3_2 + \ldots + N^3_N \right).$$

We have checked this explicitly for $N = 2$ and 4.

We now turn to the $F < 6$ theories. These can be obtained by integrating out flavors from the $F = 6$ theory. To flow to the $F = 4$ theories, we add a mass term $m(M_0)_{56}$ to the superpotentials of Eq. 6. Taking the equation of motion with respect to $(M_0)_{56}$, we obtain a quantum modified constraint for the fields of the $F = 4$ theory. More constraints follow from the equations of motion of other massive mesons $(M_i)_{56}$. These constraints are identical to the classical ones and ensure the equality of the quantum and classical moduli spaces in the limit of large expectation values. Thus, we find that the $Sp(2N)$ with $F = 4$ confines with chiral symmetry breaking. One of the classical constraints is modified quantum mechanically whereas the other $N - 1$ constraints remain unmodified. It is easy to check that the modified constraint respects all symmetries of the theory, and that the $U(1)$ charges do not allow a quantum modification of the others. The large number of constraints reflects the much richer structure of this theory compared to SUSY QCD.

There are several non-trivial consistency checks on this picture. As noted above, for $F = 4$ one can choose a $U(1)_R$ symmetry under which all the superfields of the theory are neutral. Therefore, the $U(1)_R$ symmetry remains unbroken even after chiral symmetry breaking. Since there are no non-zero R charges around, no superpotential (which has R charge two) can be generated dynamically. In the microscopic theory, both the global $U(1)^3_R$ and $U(1)_R$ anomalies are equal to $1 - 6N$. In the macroscopic theory, these anomalies are exactly matched after taking into account the N constraints on the fields M and T. The maximal unbroken subgroup of the global symmetries is $Sp(6) \times U(1)_R$. This leaves one further anomaly to check: $Sp(6)^2 U(1)_R = -4N$ in the microscopic and macroscopic theories.

We can enforce the constraints by using N Lagrange multipliers λ_i in the superpotential:

$$
\begin{aligned}
W^{Sp(4)}_{F=4} &= \lambda_1 \left(T_2 M^2_0 + \frac{1}{2} M^2_1 - \Lambda^6_{F=4} \right) + \lambda_2 M_0 M_1, \\
W^{Sp(6)}_{F=4} &= \lambda_1 \left(T^2_2 M^2_0 + T_3 M_1 M_0 - T_2 M_0 M_2 + \frac{1}{4} M^2_2 - \Lambda^8_{F=4} \right) +
\end{aligned}
$$

$$\lambda_2 \left(T_3 M_0^2 + M_1 M_2 \right) + \lambda_3 \left(-M_0^2 T_2 + M_0 M_2 + \frac{1}{2} M_1^2 \right). \quad (10)$$

Having established the superpotential for the $F = 4$ theories one can flow to $F = 2$ by adding another mass term $m(M_0)_{34}$. Integrating out the heavy fields we obtain the superpotentials for $F = 2$. We give only the $Sp(4)$ result:

$$W_{F=2}^{Sp(4)} = \frac{\Lambda_{F=2}^7 M_0}{2 T_2 M_0^2 - M_1^2}. \quad (11)$$

This superpotential can also be obtained by breaking the $Sp(4)$ to $SU(2) \times SU(2)$ with an appropriate vev of A. Instantons in the two $SU(2)$'s give this superpotential. The superpotential for larger N's are very complicated, but they are straightforward to obtain by integrating out flavors.

Finally, we can eliminate all fundamentals by adding one more mass term. In the $Sp(4)$ case, we find two branches of the $F = 0$ theory. One branch has a vanishing superpotential, while the other has a superpotential generated by gaugino condensation:

$$W = \pm \frac{\Lambda_{F=0}^4}{\sqrt{T_2}}. \quad (12)$$

On the branch where $W = 0$, the theory is in the confining phase without chiral symmetry breaking. The only global symmetry of the microscopic theory is $U(1)_R$. The T_2 operator matches all anomalies associated with this symmetry. On the branch described by the superpotential of Eq. 12 the theory has no stable vacuum.

The result for $F = 0$ is not new. $Sp(4)$ and $SO(5)$ are isomorphic, and the antisymmetric tensor of $Sp(4)$ corresponds to the fundamental of $SO(5)$. An $SO(5)$ theory with one fundamental was described in [4] and the results are in complete agreement providing a very reassuring consistency check on both the $Sp(2N)$ as well as the $SO(N)$ analysis.

3 Duality

Based on the analogy with SUSY QCD, we expect there to be a dual description for the theories with $F > 6$. Finding simple duals with only one gauge group for theories with tensor representations has proven to be very difficult, no examples of such dualities had been found prior to this work.

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After addition of a superpotential that breaks chiral symmetries and simplifies the low-energy theories by reducing the moduli space, several examples of dualities have been given [8]. The $Sp(2N)$ theory with an antisymmetric tensor has been described in Ref. [12]. Here, we tackle the theory without a superpotential. We present a dual for $F = 8$ and arbitrary N. The dual is a generalization of the known $Sp(2)$ result [5], where the antisymmetric tensor is simply a singlet.

In our dual, the meson operators $M_k = QA^kQ$ appear as fundamental fields, in addition there is a dual gauge group $Sp(2N)$ with an antisymmetric tensor, eight dual quarks and a superpotential. The dual is defined by the following field content

	$Sp(2N)$	$SU(8)$	$U(1)$	$U(1)_R$
a	$\boxminus$	1	-4	0
q	$\square$	$\bar{\square}$	$N-1$	$\frac{1}{2}$
M_0	1	$\boxminus$	$2N-2$	1
M_1	1	$\boxminus$	$2N-6$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M_{N-1}	1	$\boxminus$	$-2N+2$	1

$$(13)$$

and the superpotential

$$W = qa^{N-1}qM_0 + qa^{N-2}qM_1 + \ldots + qqM_{N-1}. \tag{14}$$

It is easy to see that the moduli spaces of the two theories agree. In the electric theory, the flat directions are described by the operators T_k and M_k, while in the magnetic theory they are given by $t_k = a^k$ and M_k. The $\widetilde{M}_k = qa^kq$ directions are however lifted by the M_k equations of motion.

A very non-trivial check on the duality is given by the 't Hooft anomaly matching conditions, which are all satisfied. Another important check is to see that this dual flows to the $F = 6$ theory after integrating out two flavors in the electric theory. In the dual, this corresponds to adding $m(M_0)_{78}$ to the superpotential of Eq. 14. The equation of motion for M_0: $m = qa^{N-1}q$ forces non-zero vevs for a and q which higgses the gauge group. The superpotential terms give rise to masses for the q's and the extra components of the M's. The superpotential of Eq. 14 does not reproduce the superpotential of Eq. 6. The missing terms are presumably generated by instantons in the broken $Sp(2N)$.

8

As a final consistency check we consider breaking the electric theory to $SU(2)^N$ by giving a vev to the A field. In the limit of large vevs this leads to N decoupled $SU(2)$'s with eight doublets each. The magnetic theory is also broken to $SU(2)^N$ by the corresponding vev of the a field, again each $SU(2)$ group has eight doublets. After defining the operator map in analogy with Eq. 9 and taking the vevs to infinity, the superpotential of Eq. 14 reduces to the correct superpotential for N factors of the dual of $SU(2)$.

We might also consider adding the operator $\text{Tr}A^k$ to our theory and its dual. We would expect to flow to the duals of the theories with the superpotential described in Ref. [12]. However, $\text{Tr}A^k$ simply maps to $\text{Tr}a^k$ in our dual, and our duals now look similar to the duals in Ref. [12], but agree only for $k = N$. For other values of k even the sizes of the dual gauge groups are different. It would be interesting to understand the connection between our duals and those of Ref. [12].

Our dual seems to contradict the expectation that the dual of a theory with arbitrary superpotential can be obtained from the dual of the theory with no superpotential. It would be very interesting to know whether this duality can be extended for $F > 8$, or if the case $F = 8$ where the electric and magnetic theories have identical gauge degrees of freedom ("self-dual") is special.

Another type of duals can be found for arbitrary values of F by using the deconfinement method of Ref. [14]. Applying the "deconfinement modulus" of Ref. [9] and choosing the additional global symmetry to be $SU(2)$ one obtains the first dual. This dual is an $Sp(F - 4) \times Sp(2N - 2)$ theory, in which the $Sp(F-4)$ gauge group has only fundamentals, while the $Sp(2N-2)$ group contains an antisymmetric tensor and $2F - 4$ fundamentals. Iterating this procedure $N-1$ times, we arrive at the gauge group $Sp(F-4) \times Sp(2F-8) \times \ldots \times Sp(N(F-4)-2)$. All the gauge groups in this chain, except for the $Sp(N(F-4)-2)$, contain an antisymmetric tensor. The $Sp(N(F-4)-2)$ group has only fundamentals and is very strongly coupled. Unfortunately, this dual description is of limited use since there is no value of F for which all gauge groups are weakly coupled.

9

4 Applications to Model Building

Now, we will use the exact results obtained in Section 2 for building models of dynamical supersymmetry breaking and of SUSY models of compositeness.

Models of Dynamical Supersymmetry Breaking

Dine, Nelson, Nir and Shirman recently noted that one can often find new models of dynamical supersymmetry breaking by reducing the gauge group of a known model [15]. Several examples of models obtained by using this method were presented in Refs. [15, 16].

We now show that one can obtain a model of dynamical supersymmetry breaking by reducing the $SU(5)$ model with a **10** and a $\bar{\mathbf{5}}$ to $Sp(4) \times U(1)$. The field content of the $Sp(4) \times U(1)$ theory is given by

$$
\begin{array}{c|cc}
 & Sp(4) & U(1) \\
\hline
A & \boxminus & 2 \\
Q_1 & \square & -3 \\
Q_2 & \square & -1 \\
S_1 & 1 & 2 \\
S_2 & 1 & 4
\end{array}
\tag{15}
$$

We add the tree-level superpotential

$$
W_{tree} = Q_1 Q_2 S_2 + Q_1 A Q_2 S_1.
\tag{16}
$$

The $Sp(4)$ invariants and their $U(1)$ charges are

$$
\begin{array}{c|c}
 & U(1) \\
\hline
T_2 = A^2 & 4 \\
M_0 = Q_1 Q_2 & -4 \\
M_1 = Q_1 A Q_2 & -2 \\
S_1 & 2 \\
S_2 & 4
\end{array}
\tag{17}
$$

The S_1 and S_2 equations of motion set M_0 and M_1 to zero, and the $U(1)$ charges of the remaining $Sp(4)$ invariants are all of the same sign. Therefore, all classical flat directions are lifted by the superpotential of Eq. 16.

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The full superpotential also contains a term generated by the $Sp(4)$ dynamics, as described in Eq. 11:

$$W = Q_1 Q_2 S_2 + Q_1 A Q_2 S_1 + \frac{\Lambda^7 (Q_1 Q_2)}{2(A^2)(Q_1 Q_2)^2 - (Q_1 A Q_2)^2}. \qquad (18)$$

The tree-level superpotential preserves a non-anomalous $U(1)_R$ symmetry, which has to be broken because of the presence of the dynamically-generated term. Therefore, one expects supersymmetry to be dynamically broken, and indeed the equations of motion are contradictory.

One can also built models of dynamical supersymmetry breaking by making use of quantum modified moduli spaces [17]. Consider for example an $Sp(4)$ theory with $F = 4$ and additional singlets S_i coupled via the following superpotential

$$W_{tree} = S_1 M_0 + S_2 M_1 + S_3 T_2. \qquad (19)$$

The fields S_1 and S_2 transform as antisymmetric tensors under the $SU(4)$ flavor symmetry. The equations of motion for the singlet fields set $M_0 = M_1 = T_2 = 0$. Such a solution does not lie on the quantum modified moduli space described by Eq. 10, therefore supersymmetry is dynamically broken in this theory.

Composite Model Building

These new results may have interesting applications for SUSY models of compositeness [18, 19]. For example, it is an intriguing possibility that the three families of the standard model are composite and arise after confinement of a preonic $Sp(6)$ theory with an antisymmetric tensor and $F = 6$ fundamentals. As discussed above, the infrared spectrum of this theory contains the three meson fields $M_0 = QQ$, $M_1 = QAQ$, and $M_2 = QA^2Q$ which transform as antisymmetric tensors of the flavor symmetry. Thus, if an $SU(5)$ subgroup of the $SU(6)$ flavor symmetry was weakly gauged, one would find three generations of **10**'s of $SU(5)$ after the Sp confinement. Alternatively, one might just gauge an $SU(3) \times SU(2) \times U(1)$ subgroup to obtain the supersymmetric standard model.

To illustrate this idea we present a simple toy model that reproduces the particle content of a SUSY $SU(5)$ GUT plus two vector like pairs of $\mathbf{5} + \bar{\mathbf{5}}$.

The model consists of the fields given in the following table:

	$Sp(6)$	$SU(5)$	$SU(6)$
A	$\boxplus$	1	1
Q	$\square$	$\square$	1
Q'	$\square$	1	1
$\overline{Q}$	1	$\overline{\square}$	$\square$

$$(20)$$

After confinement of the $SP(6)$ we obtain

	$SU(5)$	$SU(6)$
$M_0,\ M_1,\ M_2$	$\boxplus$	1
$H_0,\ H_1,\ H_2$	$\square$	1
$T_2,\ T_3$	1	1
$\overline{Q}$	$\overline{\square}$	$\square$

$$(21)$$

and the non-perturbatively generated superpotential

$$
\begin{aligned}
W \;=\;\; & H_0\left(\frac{T_2^2}{\Lambda^2}M_0^2 - \frac{T_3}{\Lambda}M_0 M_1 + \frac{T_2}{\Lambda}M_0 M_2 - M_2^2\right) + \\
& H_1\left(-\frac{T_3}{\Lambda}M_0^2 + M_1 M_2\right) + H_2\left(\frac{T_2}{\Lambda}M_0^2 - M_0 M_2 + M_1 M_1\right). \quad (22)
\end{aligned}
$$

When the composite field H_0 is identified with H_U of the supersymmetric standard model this results in the following Yukawa coupling matrix for the up-type quarks is obtained:

$$
Y_U \sim \begin{pmatrix} T_2^{\,2}/\Lambda^2 & T_3/\Lambda & T_2/\Lambda \\ T_3/\Lambda & 0 & 0 \\ T_2/\Lambda & 0 & 1 \end{pmatrix}
\tag{23}
$$

Assume that the vacuum expectation values of the composite T_i are somewhat smaller than the scale Λ where the $SP(6)$ confines, the theory generates a hierarchical structure of Yukawa couplings dynamically. Such vevs for the T_i's could be enforced by adding a tree-level superpotential.

This way the hierarchical structure of Yukawa couplings that is so difficult to obtain in conventional models arises automatically as a consequence of the strong preon dynamics! From a low-energy point of view the origin of the hierarchical structure can be understood in terms of a "horizontal" $U(1)$

symmetry under which the generations transform with different charges. In the ultraviolet this "horizontal" symmetry is simply the anomaly-free $U(1)$. The difference in charges between the generations reflects the fact that the M_i contain different numbers of the preon field A.

Down quark and lepton Yukawa couplings arise from higher-dimensional operators that have to be present before the confinement of the Sp gauge group. Requiring that the superpotential respects the above-mentioned $U(1)$ symmetry, one finds down Yukawa couplings with a somewhat smaller hierarchy than in the up sector. This is nice because the quark mass ratios in the down sector are smaller than in the up sector. Obviously, this toy model is too simple to account for all details of the standard model. This is left for a future investigation.

5   Conclusions

We have investigated the non-perturbative behavior of SUSY $Sp(2N)$ gauge theories containing an antisymmetric tensor. The theory with $F = 6$ confines without chiral symmetry breaking. The low-energy degrees of freedom are the mesons $M_k = QA^kQ$ and $T_k = \text{Tr}A^k$. They interact via a confining superpotential, which was determined exactly by demanding consistency with the classical constraints. Starting from the $F = 6$ theory we obtain non-trivial results for the theory with less flavors:

– The $F = 4$ theory confines with chiral symmetry breaking; an interesting result is that there are N constraints, one of which is modified quantum mechanically.

– The $F = 2$ theory has an instanton-generated superpotential and a runaway vacuum.

– The $F = 0$ theories have two disconnected branches of vacua: one with no superpotential and a moduli space of vacua, and one with a runaway vacuum.

We have constructed a dual description to the $F = 8$ theory, which is the first of this kind in the literature. It is not obvious how this dual relates to the known duals of theories with added superpotential. It would be very interesting to understand whether this dual for $F = 8$ is due to a fortunate coincidence or it can be generalized to larger values of F.

We also presented two interesting applications of our results to model

building. One application is to construct models of dynamical supersymmetry breaking. Another is for building composite models. One can embed the supersymmetric standard model into an $Sp(6) \times SU(5)$ gauge theory, where $Sp(6)$ has an antisymmetric tensor and $F = 6$. Confinement in $Sp(6)$ results in composite quarks and leptons. A mass hierarchy for the standard model particles arises naturally as a result of the confining superpotential of $Sp(6)$.

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